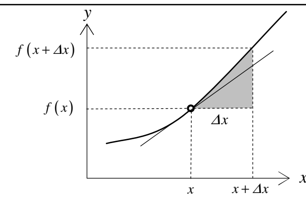
Differentiation

Derivative

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$



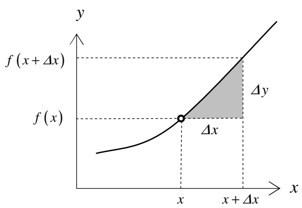
$\frac{df}{dx}$ *slope of the tangent line*

$\frac{df}{dx} > 0$ *if* $f(x)$ *is increasing*

$\frac{df}{dx} < 0$ *if* $f(x)$ *is decreasing*

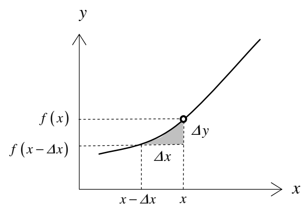
Approximations of the Derivative

forward difference



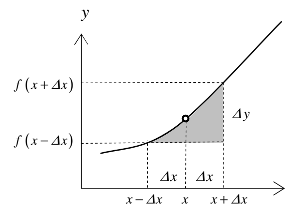
$$f'(x) \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

back difference



$$f'(x) \approx \frac{f(x) - f(x-\Delta x)}{\Delta x}$$

central difference



$$f'(x) \approx \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$$

Linear Ordinary Differential Equations

1st order ODE :

$$\frac{dy}{dx} + P(x)\, y = Q(x)$$

***I**nitial **V**alue **P**roblem:*

$$y(x_0) = y_0$$

Integrating Factor :

$$\mu(x) = e^{\int P(x)dx} \quad or \quad \mu(x) = e^{\int_{x_0}^{x} P(x)dx}$$

General Solution :

$$y = \frac{c}{\mu(x)} + \frac{1}{\mu(x)} \int \mu(x) Q(x)\, dx$$

Solution of IVP :

$$y = \frac{\mu(x_0)}{\mu(x)} y_0 + \frac{1}{\mu(x)} \int_{x_0}^{x} \mu(x') Q(x')\, dx'$$

Constant Coefficient $P(x) = a$ **:**

$$y = y_0 e^{a(x_0 - x)} + e^{-ax} \int_{x_0}^{x} e^{ax'} Q(x')\, dx'$$

Homogeneous ODE $(Q = 0)$ **:**

$$\frac{dy}{dx} + P(x)\, y = 0$$

General Solution :

$$y = \frac{c}{\mu(x)}$$

Solution of IVP :

$$y = \frac{\mu(x_0)}{\mu(x)} y_0$$

Constant Coefficient $P(x) = a$ **:**

$$y = y_0 e^{a(x_0 - x)}$$

2nd order homogeneous ODE with constant coefficients :

$$\frac{d^2 y}{dx^2} - m^2 y = 0 \qquad x \in (0, L)$$

General Solution can be in one of the following forms :

$$y = c_1 e^{-mx} + c_2 e^{mx}$$

$$y = c_1 e^{-m(x-L)} + c_2 e^{m(x-L)}$$

$$y = c_1 \sinh(mx) + c_2 \cosh(mx)$$

$$y = c_1 \sinh\left[m(x-L)\right] + c_2 \cosh\left[m(x-L)\right]$$

Boundary Value Problem *(there are 3 types of the boundary conditions):*

$y(0) = y_0$	$y'(0) = f_0$	$-ky'(0) + hy(0) = f_0$
$y(L) = y_L$	$y'(L) = f_L$	$ky'(L) + hy(L) = f_L$

(one boundary condition has to be set at x=0, and one boundary condition at x=L)

Solution of BVP :

find the coefficients c_1 *and* c_2 *such that solution satisfies the boundary conditions*